Baryogenesis and phase transition in the standard model

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ABSTRACT

The sphaleron type solution in the electroweak theory, generalized to include the dilaton field, is examined. The solutions describe both the variations of Higgs and gauge fields inside the sphaleron and the shape of the dilaton cloud surrounding the sphaleron. Such a cloud is large and extends far outside. These phenomena may play an important role during the baryogenesis which probably took place in the Early Universe.

1. INTRODUCTION

In this paper the electroweak theory will be extended by the inclusion of dilatonic field. The Glashow-Weinberg-Salam dilatonic model with $SU_L(2) \times U_Y(1)$ symmetry is described by the lagrangian

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}e^{2\varphi(x)/f}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}e^{2\varphi(x)/f}B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{1}{4}\partial_\mu\varphi\partial^\mu\varphi + (D_\mu H)^+D^\mu H - U(H,\varphi) \end{aligned} \tag{1}$$

with the $SU_L(2)$ field strength tensor $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon_{abc}W^b_\mu W^{c\nu}$ and the $U_Y(1)$ field tensor $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The covariant derivative is given by $D_\mu = \partial_\mu - \frac{1}{2}igW^a_\mu\sigma^a - \frac{1}{2}g^{'}YB_\mu$, where B_μ and $W_\mu = \frac{1}{2}W^a_\mu\sigma^a$ are respectively local gauge fields associated with $U_Y(1)$ and $SU_L(2)$ symmetry groups. Y denotes the hypercharge. The gauge group is a simple product of $U_Y(1)$ and $SU_L(2)$ hence we have two gauge couplings g and g'. The generators of gauge groups are: a unit matrix for $U_Y(1)$ and Pauli matrices for $SU_L(2)$. In the simplest version of the standard model a doublet of Higgs

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fields is introduced $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix}$, with the Higgs potential

$$U(H^+, H, \varphi) = \lambda \left(H^+ H - \frac{1}{2} v_0^2 e^{2\varphi(x)/f} \right)^2 \tag{2}$$

where $f = 10^7 GeV$ [1] determines the dilaton scale, $v_0 = 250 GeV$, and v is the vacuum expectation value for the Higgs field. The form of the potential (2) leads to vacuum degeneracy and to nonvanishing vacuum expectation value of the Higgs field and consequently to fermion and boson masses. In the process of spontaneous symmetry breaking the Higgs field acquires nonzero mass [2, 3].

2. THE DILATONIC SPHALERON

Let us now consider the sphaleron type solution in the electroweak theory with dilatons. The sphaleron may be interpreted as inhomogeneous bosons condensate $d(x), W_\mu^a(x)$. Let us assume for simplicity that $g' = 0$. (In [4] the sphaleron theory was also considered also for $g' \neq 0$.) Now we make the following anzatz for the sphaleron Higgs field

$$H = \frac{1}{\sqrt{2}} v_0 U(x) h(r) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3}$$

where $U(x) = i \sum \sigma^a n^a$ and $n^a = \frac{r^a}{r}$ describe the *hedgehog* structure. This produces a nontrivial topological charge of the sphaleron. The topological charge is equal to the Chern-Simons number. Such a *hedgehog* structure determines the asymptotic shape of the sphaleron with gauge fields different from zero

$$W_i^a = \epsilon_{aij} n^j \frac{1 - s(r)}{gr}. \tag{4}$$

Spherical symmetry is assumed for the dilaton field $d(r)$, as well as for the Higgs field $h(r)$ and the gauge field $s(r)$, leading to the following effective lagrangian

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} f^2 \left(\frac{d'(r)}{d(r)} \right)^2 - \frac{1}{2} v_0^2 h'(r)^2 - \frac{1}{4} \lambda v_0^4 (d(r)^2 - h(r)^2)^2 \\ &- \frac{1}{4r^2} v_0^2 (3 - s(r))^2 h(r)^2 - \frac{1}{2g^2 r^2} d(r)^2 \left(\frac{1}{r^2} (3 - 4s(r) + s(r)^2)^2 + 2s'(r)^2 \right) \\ &- \frac{1}{4} C v_0^4 h(r)^4 \left(ln(h(r)^2) - \frac{1}{2} \right) \end{aligned} \tag{5}$$

Then we switch to dimensionless variables $x = M_W r = r/r_W$, where $M_W^2 = \frac{1}{4}g^2 v_0^2 \sim 80GeV$, $r_W = \frac{1}{M_W} \sim 10^{-18}cm$. The resulting Euler-Lagrange equations are following: $s(x)$ function, which describes the gauge field in the electroweak theory and satisfies the equation

$$\begin{aligned} s''(x) &+2\frac{d(x)'}{d(x)}s'(x) + \frac{h(x)^2}{d(x)^2}(3 - s(x)) \\ + &\frac{1}{x^2}(1 - s(x))(2 - s(x))(3 - s(x)) = 0, \end{aligned} \quad (6)$$

and $h(x)$ function describing the Higgs field in our theory which satisfies the equation:

$$\begin{aligned} h''(x) &+\frac{2\,h'(x)}{x} + \frac{1}{2}\frac{M_H^2}{M_W^2}(d(x)^2 - h(x)^2)h(x) \\ - &\frac{8C}{g^2}h(x)^3 ln(h(x)^2) - \frac{1}{2x^2}(s(x) - 3)^2 h(x) = 0, \end{aligned} \quad (7)$$

where $M_H^2 = 2\lambda v_0^2$ determines the Higgs mass. The $d(x)$ function describing the dependence of a dilaton field on x in extended electroweak theory obeys the equation:

$$\begin{aligned} d''(x) &+\frac{2}{x}d'(x) - \frac{d'(x)^2}{d(x)} + \frac{M_H^2}{g^2 f^2}\left(\frac{1}{x^4}(s(x) - 3)^2(s(x) - 1)^2 d(x)^3\right. \\ + &\left. 2\lambda {v^4}_0(d(x)^2 - h(x)^2)d(x)^3 + \frac{4}{g^2x^2}d(x)^3 s'(x)^2\right) = 0 \end{aligned} \quad (8)$$

In the last equation we have a dimensionless constant $(\frac{M_W}{gf})^2 \sim 10^{-9}$. This practically means that the dilaton field is a free field. The simplest solutions $h(x) = 1$ (shown in Fig.1), $s(x) = 3$ (Fig.2), $d(x) = 1$ (Fig.3) are global ones corresponding to the vacuum with broken symmetry in the standard model. It is obvious that far from the center of the sphaleron our solutions should describe the normal broken phase which is very well known from the standard model. Knowing the asymptotic solutions we are able to construct a two-parameter family of solutions (for details see [5]):

$$s(x) = 1 + 2\tanh^2(tx) \quad (9)$$

$$h(x) = \tanh(ux) \quad (10)$$

$$d(x) = a + (1-a)\tanh^2(kx) \tag{11}$$

where t, u, a, k are parameters to be determined by the variational procedure. The relevant values of the parameters are those which minimize the energy. For example, with the standard values of $M_W = 80.6GeV$, $M_Z = 91.16GeV$, $M_H = 350GeV$ we found the numeric solutions t, u, k, as functions depending on the initial conditions of the dilaton field $d(0) = a$ in the center of the sphaleron. Our solutions describe both the behavior of Higgs field and gauge field inside the sphaleron and the shape of the dilaton cloud surrounding the sphaleron. Such a cloud is large and extends far outside the sphaleron. The sphalerons are created during the first order phase transition in the expanding universe as inhomogeneous solutions of the motion equations. These phase transition bubbles, which probably took place in the early universe, break the CP and C symmetry on their walls and can cause the breaking of baryonic symmetry. Detailed consideration of this problem will be the subject of a separate paper.

3. CONCLUSIONS

Numerical solutions suggest that sphaleron possess an 'onionlike' structure. In the small inner core the scalar field is decreasing with global gauge symmetry restoration $SU(2) \times U(1)$. In the middle layer the gauge field undergoes sudden change. The sphaleron coupled to dilaton field has also an outer shell, where dilaton field changes drastically. The spherically symmetric dilaton solutions coupled to the gauge field or gravity are interesting in their own rights and may further influence the monopole catalysis of baryogenesis induced by sphaleron.

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FIGURE CAPTIONS

Figure 1. The dependence of the Higgs field $h(x)$ on x.
Figure 2. The dependence of the gauge field $s(x)$ on x.
Figure 3. The dependence of the dilaton field $d(x)$ on x.

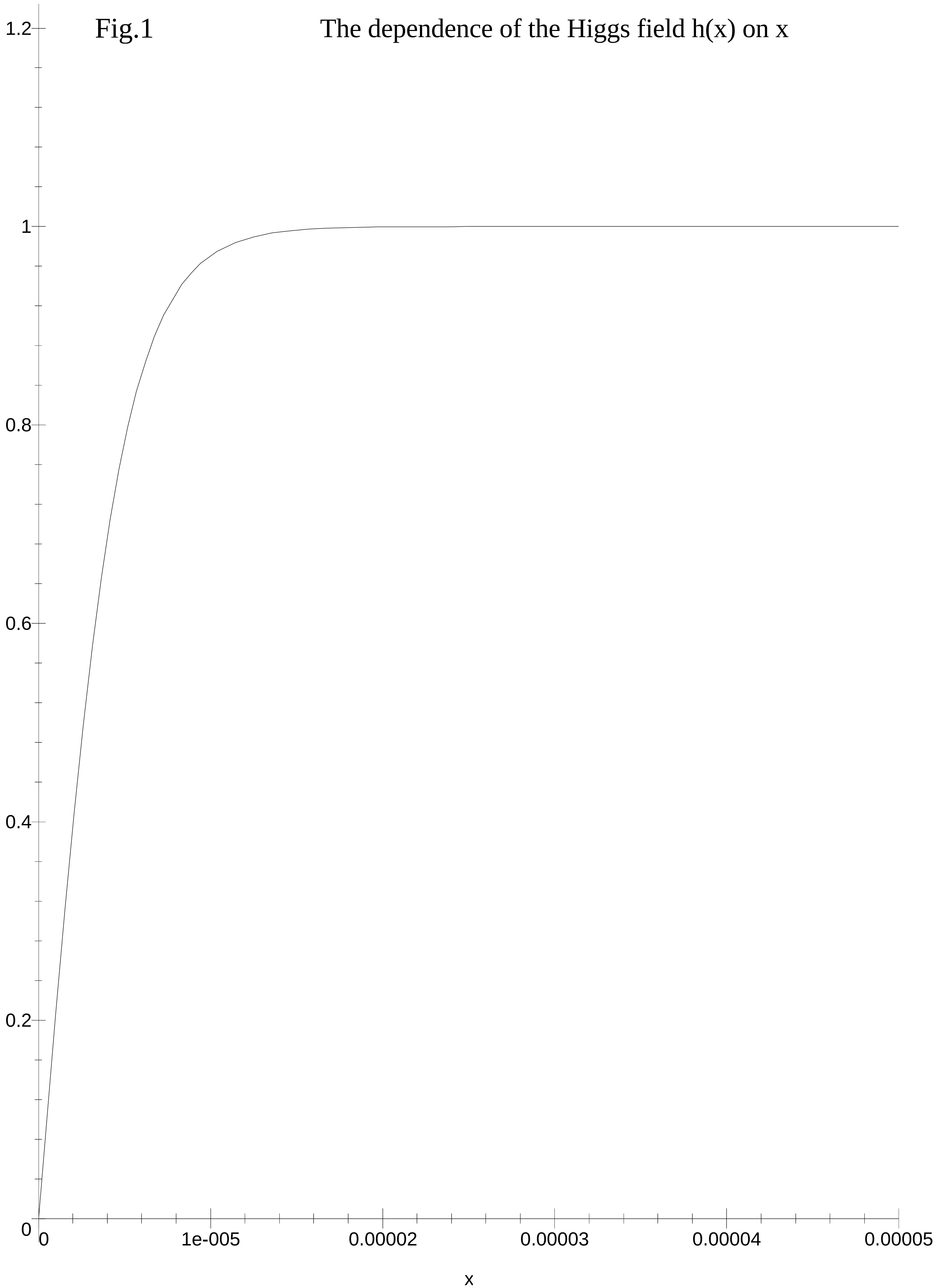
Fig.1
The dependence of the Higgs field h(x) on x
1.2
1
0.8
0.6
0.4
0.2
0
0
1e-005
0.00002
0.00003
0.00004
0.00005
x

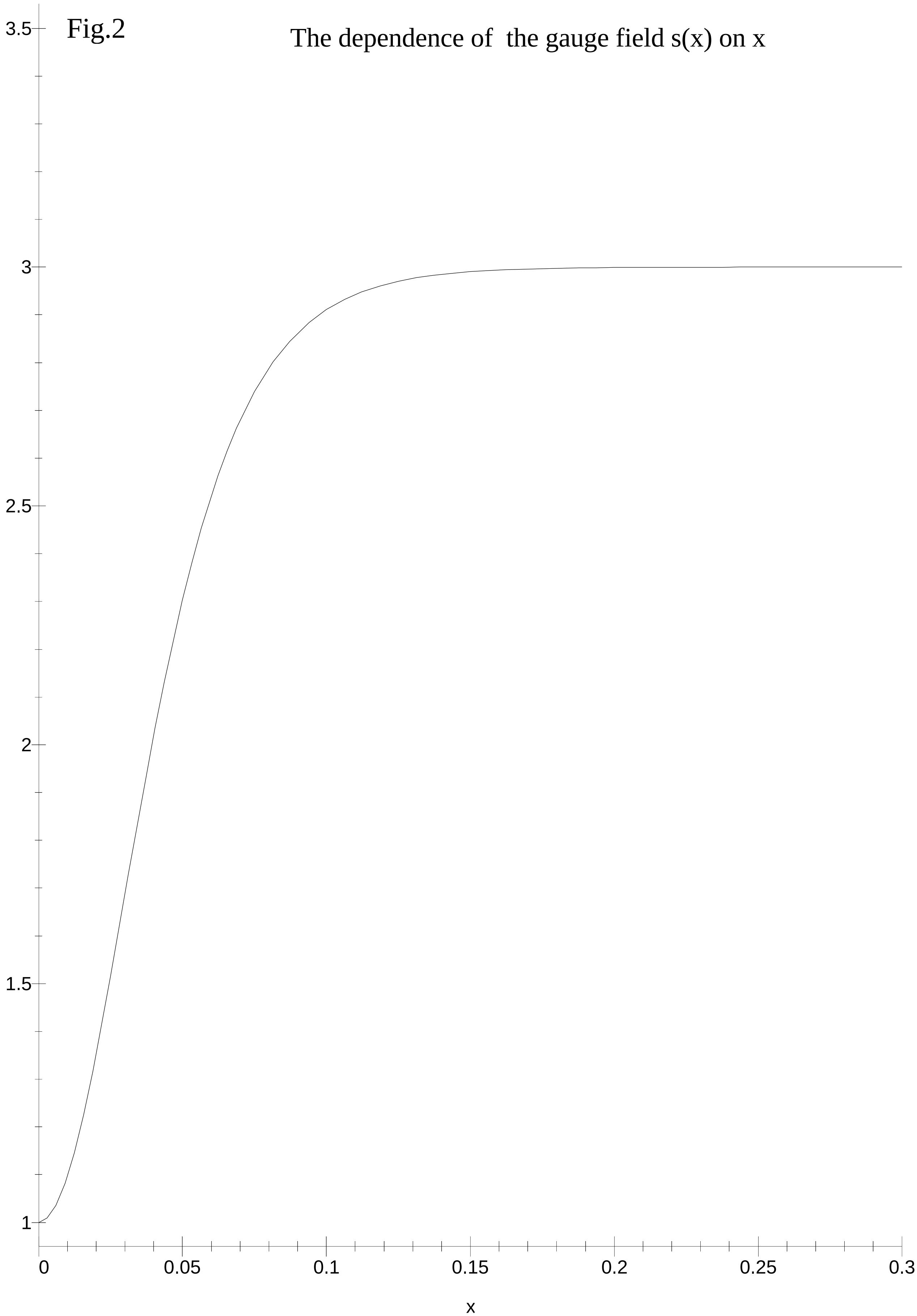
Fig.2
The dependence of the gauge field s(x) on x
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
0.3
x

Fig.3 The dependence of the dilaton field d(x) on x